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# Visualizations to Summarize Search Behavior

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# Introduction

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- Backtrack search when solving a constraint satisfaction problem (CSP) suffers from thrashing
- Enforcing a consistency property reduces thrashing at the cost of further processing
- The tradeoff of running a higher-level consistency (HLC) is poorly understood.
- We propose to summarize search

# Background

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- A constraint satisfaction problem (CSP) consists of a set of variables, the variables' domains, and constraints.
- A solution is a domain to variable assignment that satisfies all constraints.
- Backtrack search is only sound and complete method of solving a CSP.

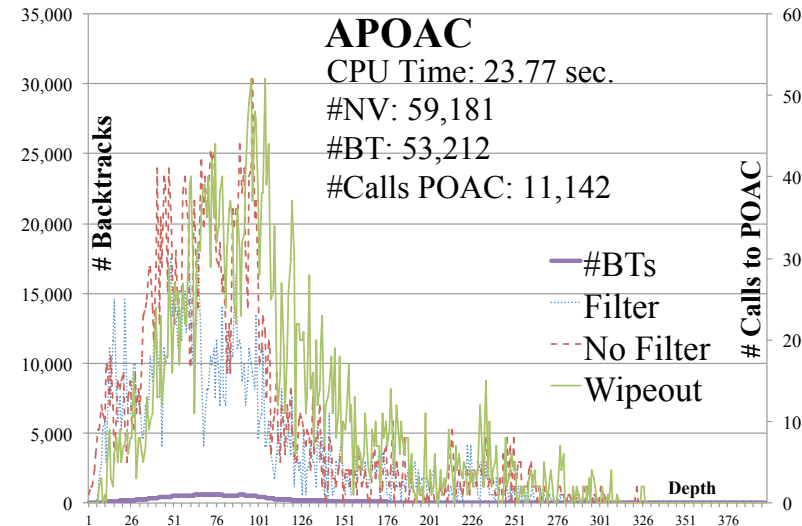
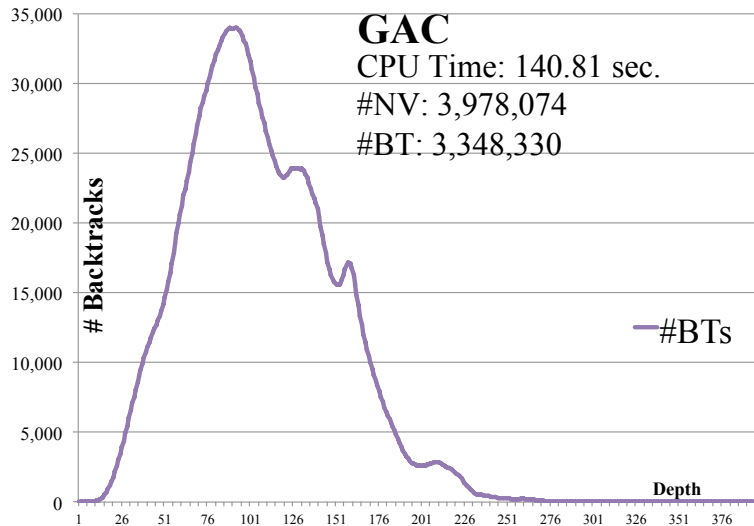
# Background

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- Thrashing is the main malady of search
  - Repeatedly performing same logic
- Applying a consistency algorithm after variable instantiation reduces thrashing
- Past visualizations focus on debugging and inspecting search
  - Detect thrashing through isomorphic subtrees
  - Debug individual issues by investigating the state of variables, propagators, etc.

# Background

- Woodward et al. propose to measure:
  - thrashing with Backtracks per Depth (BpD)
  - HLC cost with consistency calls per depth (CpD)



# Contributions

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1. Criteria for computing distance between two time samples based on the BpD
2. A clustering technique for summarizing search into a history of qualitatively distinct regimes
3. A new visualization that examines the behavior of variable ordering heuristics

# Outline

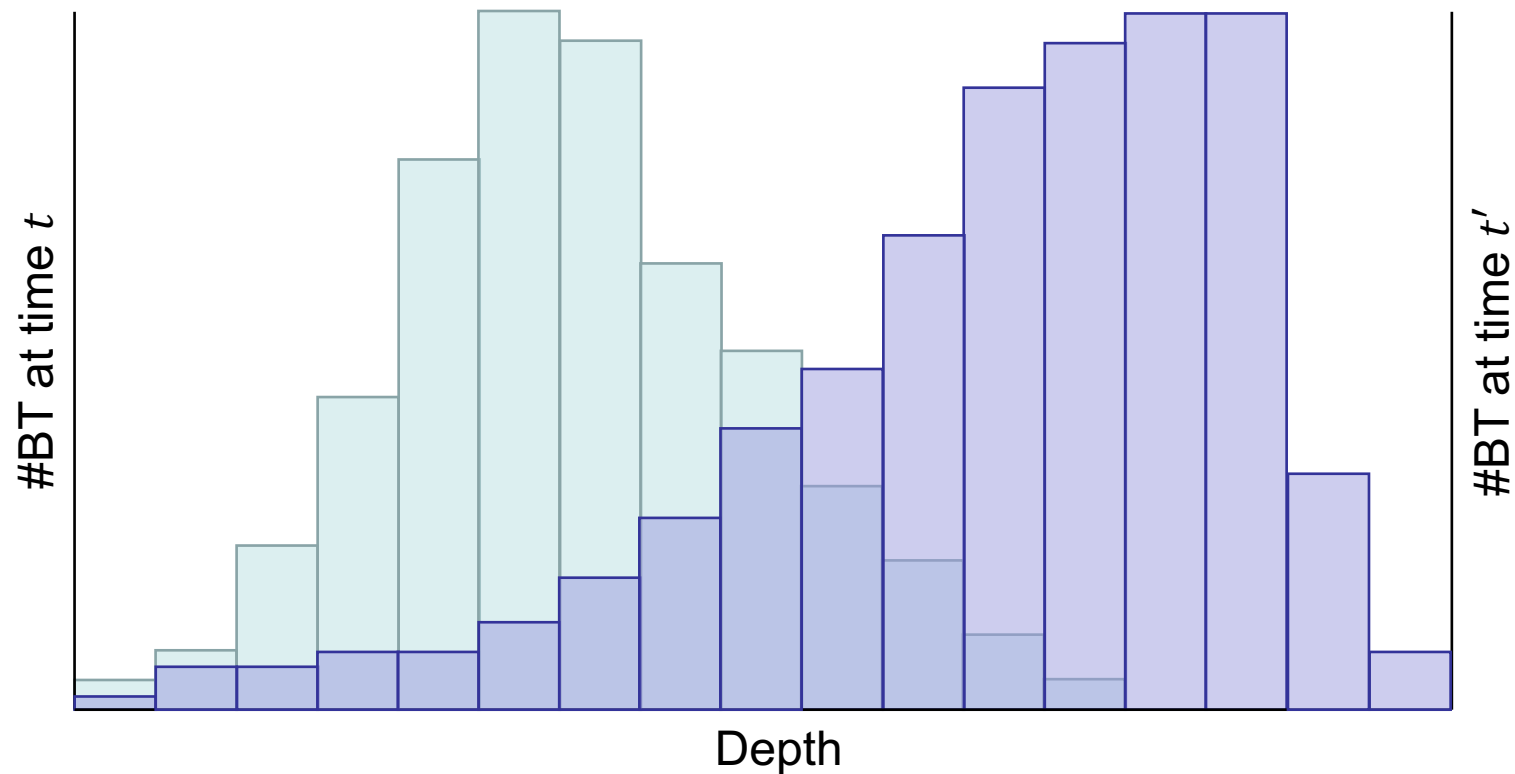
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1. Introduction
2. Background
3. Contributions
4. **Outline**
5. Distance between timestamps
6. Summarizing search history
7. Example: Analyzing structure
8. Visualizing variable ordering
9. Example: Variable ordering
10. Conclusions and discussion

# Distance between timestamps

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- Design goal: capture the “shape” change





# Distance between timestamps

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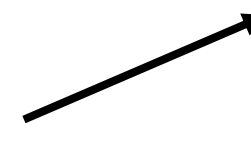
- Adapt Kulbak-Liebler Divergence
- BpD distribution:
  - Use additive smoothing to account for zero-probability intervals


$$\hat{p}(d, t) = \frac{\text{BT}(d, t) + \alpha}{\sum_{d \in D} \text{BT}(d, t) + \alpha d_{\max}}$$

# Distance between timestamps

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$$\text{div}(t, t') = \max \left( \sum_{d \in D} \hat{p}(d, t) \log \left( \frac{\hat{p}(d, t)}{\hat{p}(d, t')} \right), \right.$$

$KL(t, t')$    $\sum_{d \in D} \hat{p}(d, t') \log \left( \frac{\hat{p}(d, t')}{\hat{p}(d, t)} \right)$

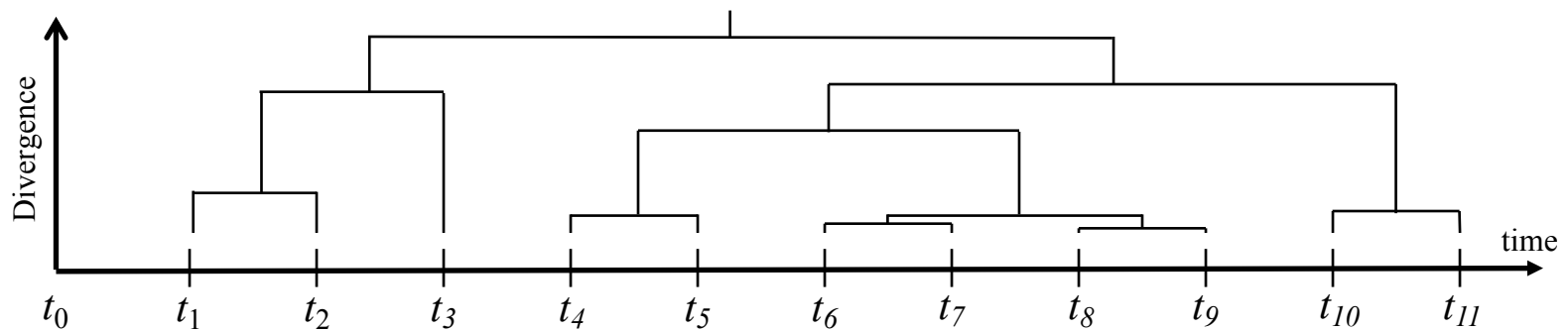
$KL(t', t)$    $\sum_{d \in D} \hat{p}(d, t) \log \left( \frac{\hat{p}(d, t)}{\hat{p}(d, t')} \right)$

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# Summarizing search history

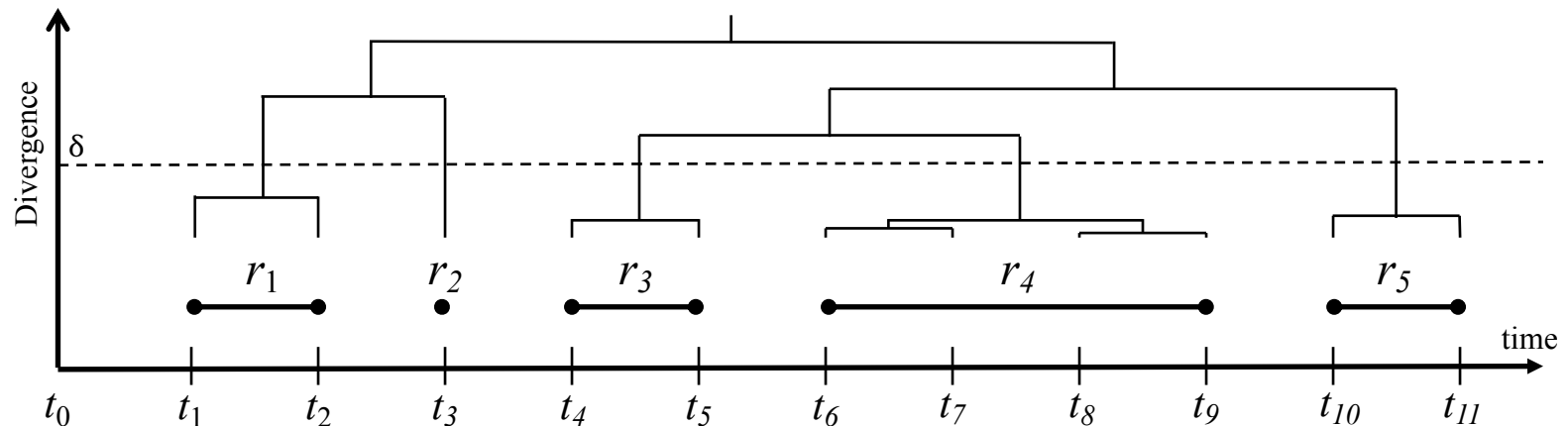
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- Create a clustering tree
  - Use agglomerative, hierarchical clustering
  - Only merge temporally adjacent clusters
  - Each cluster's representative is the middle of the interval of included timestamps



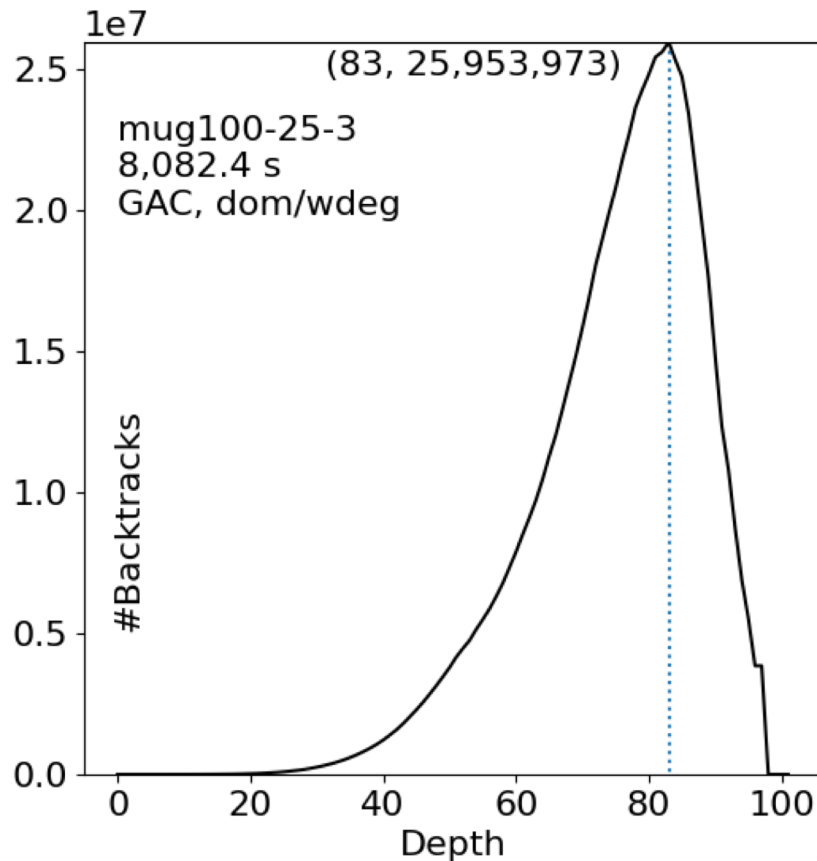
# Summarizing search history

- Create a summarization tree
  - Cut the tree at a user-defined  $\delta$  or
  - Cut the tree to include a  $n$  regimes

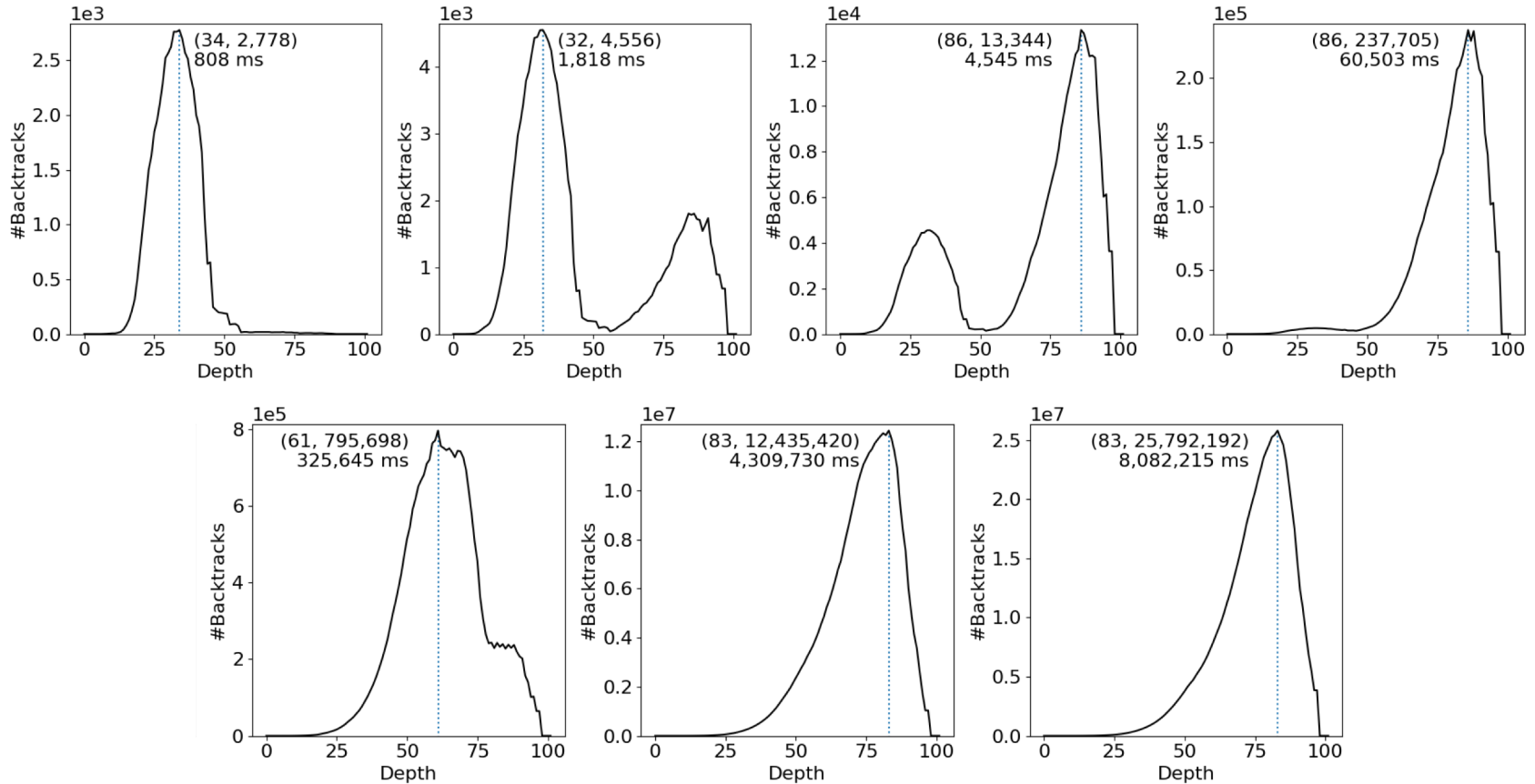


# Example: Analyzing structure

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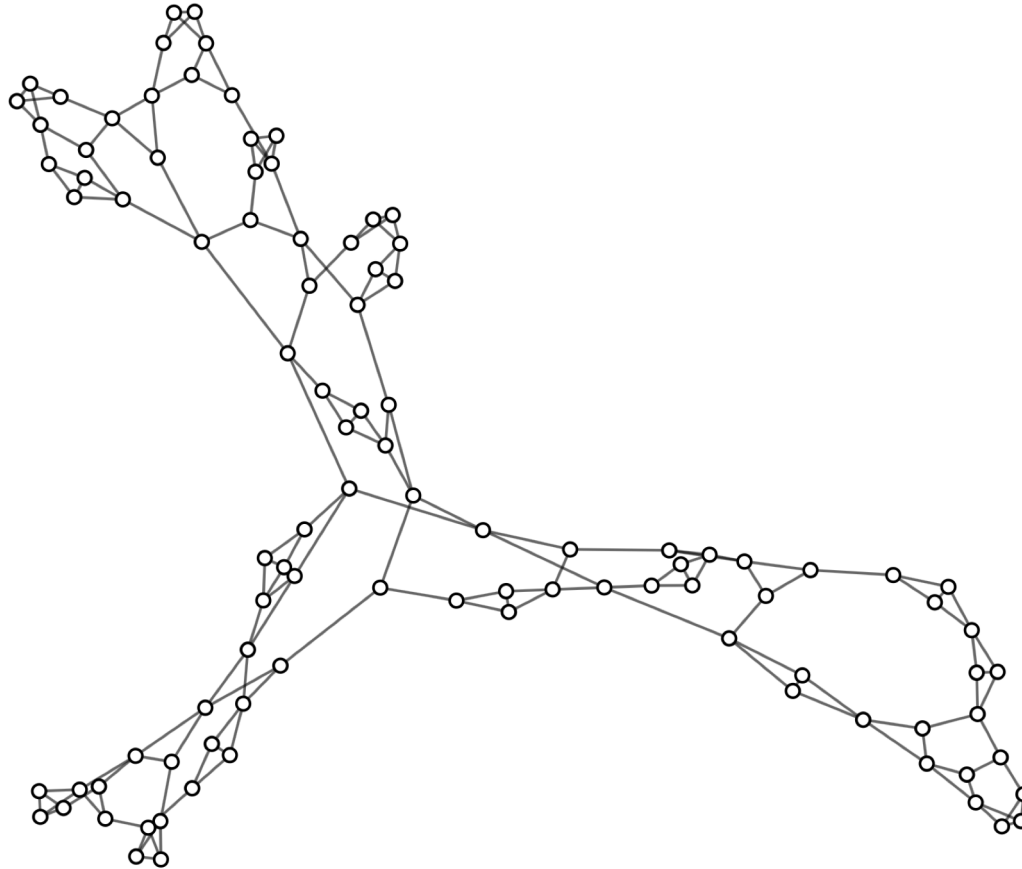
# Example: Analyzing structure



0.1-summarization of GAC on MUG100-25-3

# Example: Analyzing structure

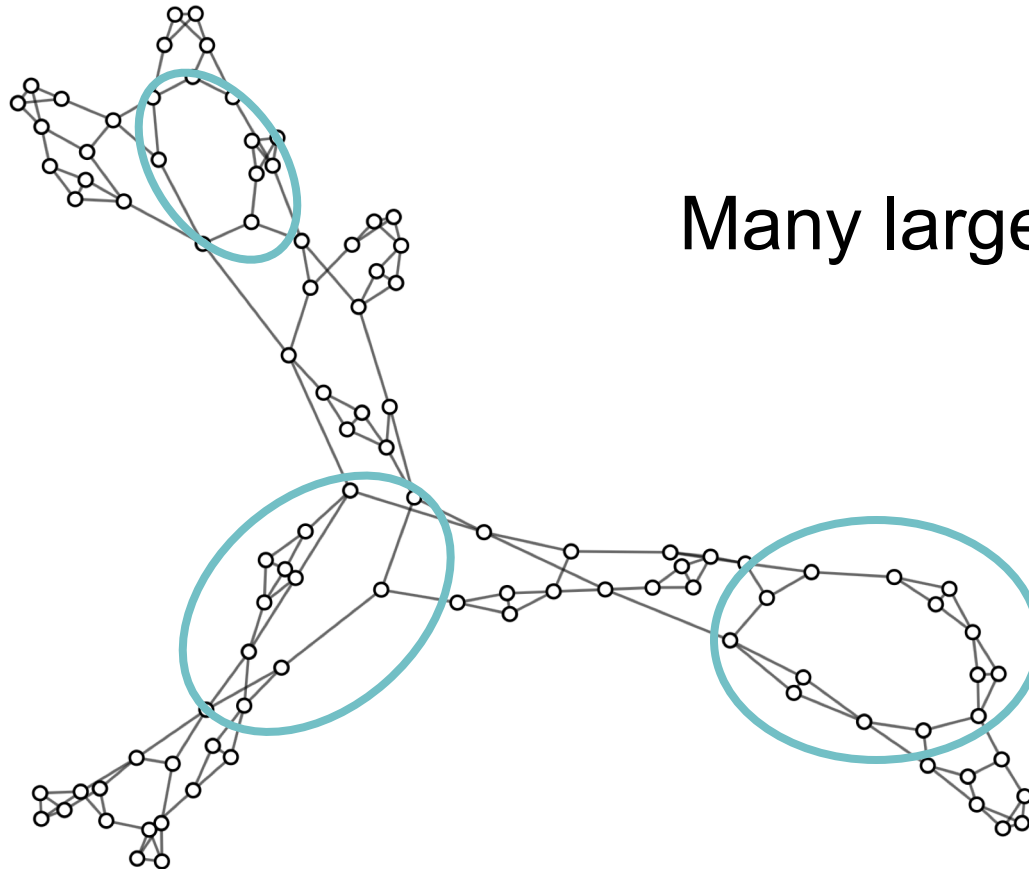
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Primal graph of MUG100-25-3

# Example: Analyzing structure

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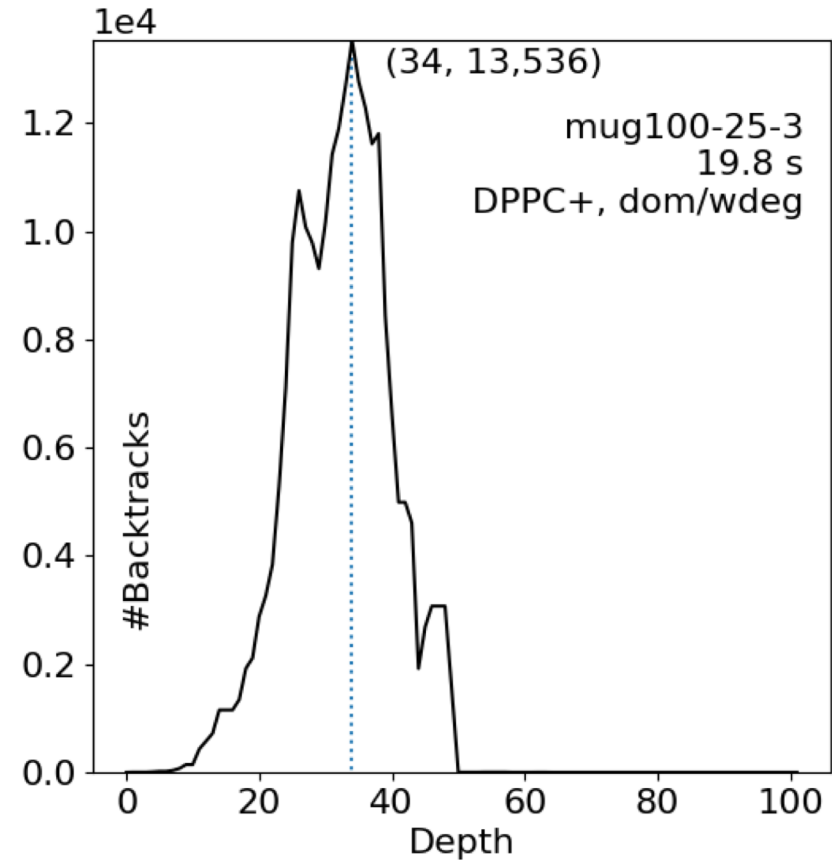
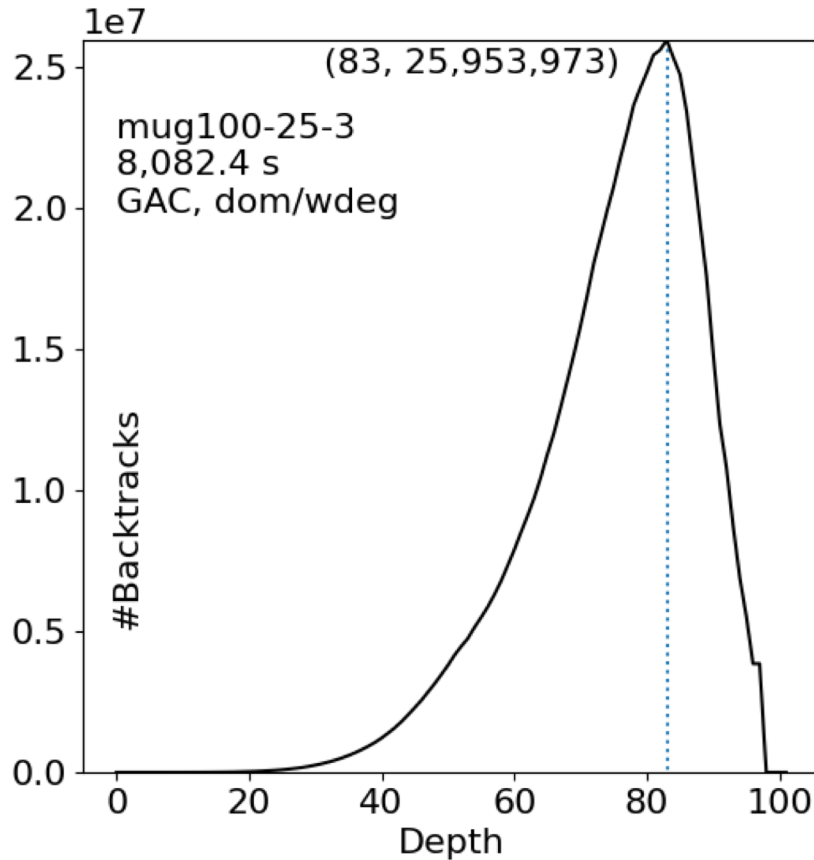


Many large cycles

Primal graph of MUG100-25-3



# Example: Analyzing structure



BpD of GAC and DPPC+ on MUG100-25-3

# Visualizing variable ordering

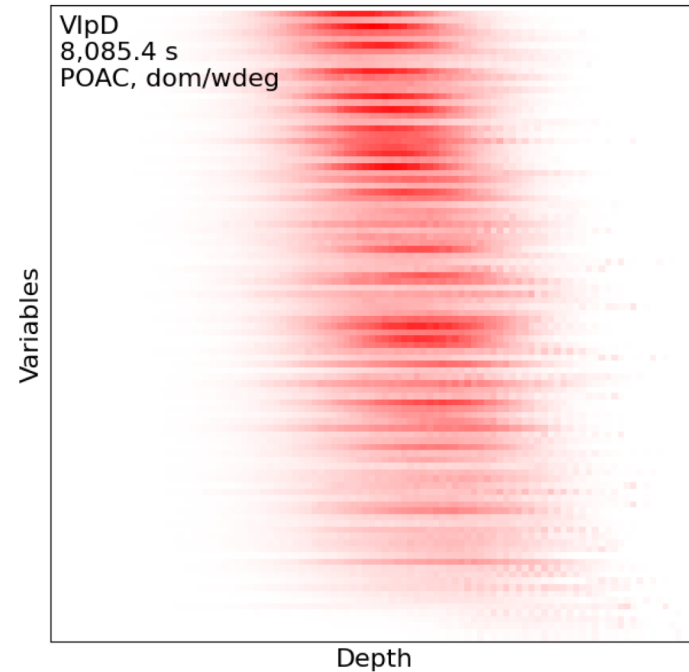
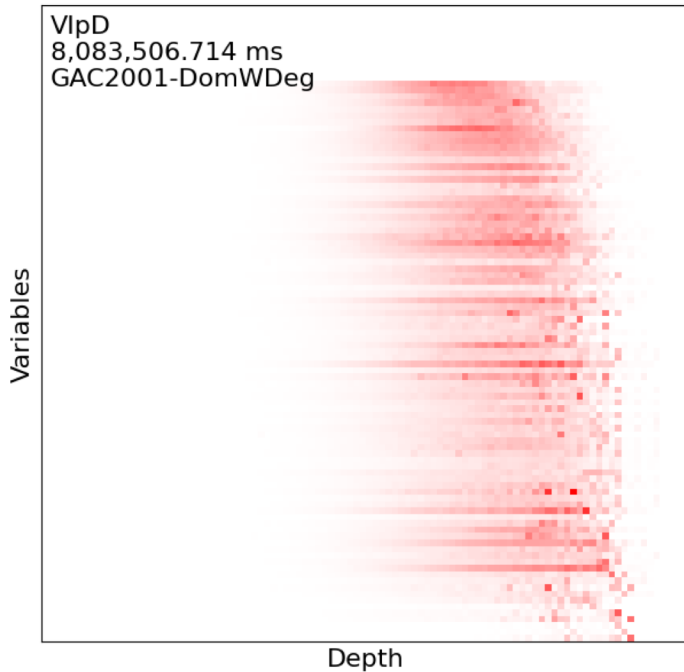
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- Variable Instantiations per Depth (VIpD)
  - $I(v, d, t)$  is the number of instantiations of variable  $v$  at depth  $d$  and time  $t$
- Order the variables of the VIpD according to each variable's weighted depth:

$$d_w(v, t) = \frac{\sum_{d \in D} I(v, d, t) \cdot d}{\sum_{d \in D} I(v, d, t)}$$

# Example: Variable ordering

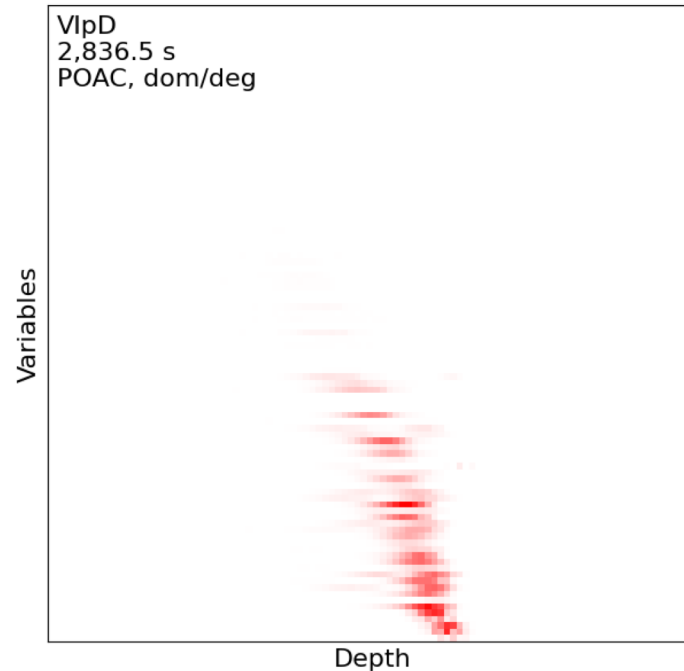
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VlpD of GAC and POAC with dom/wdeg on MUG100-1-3

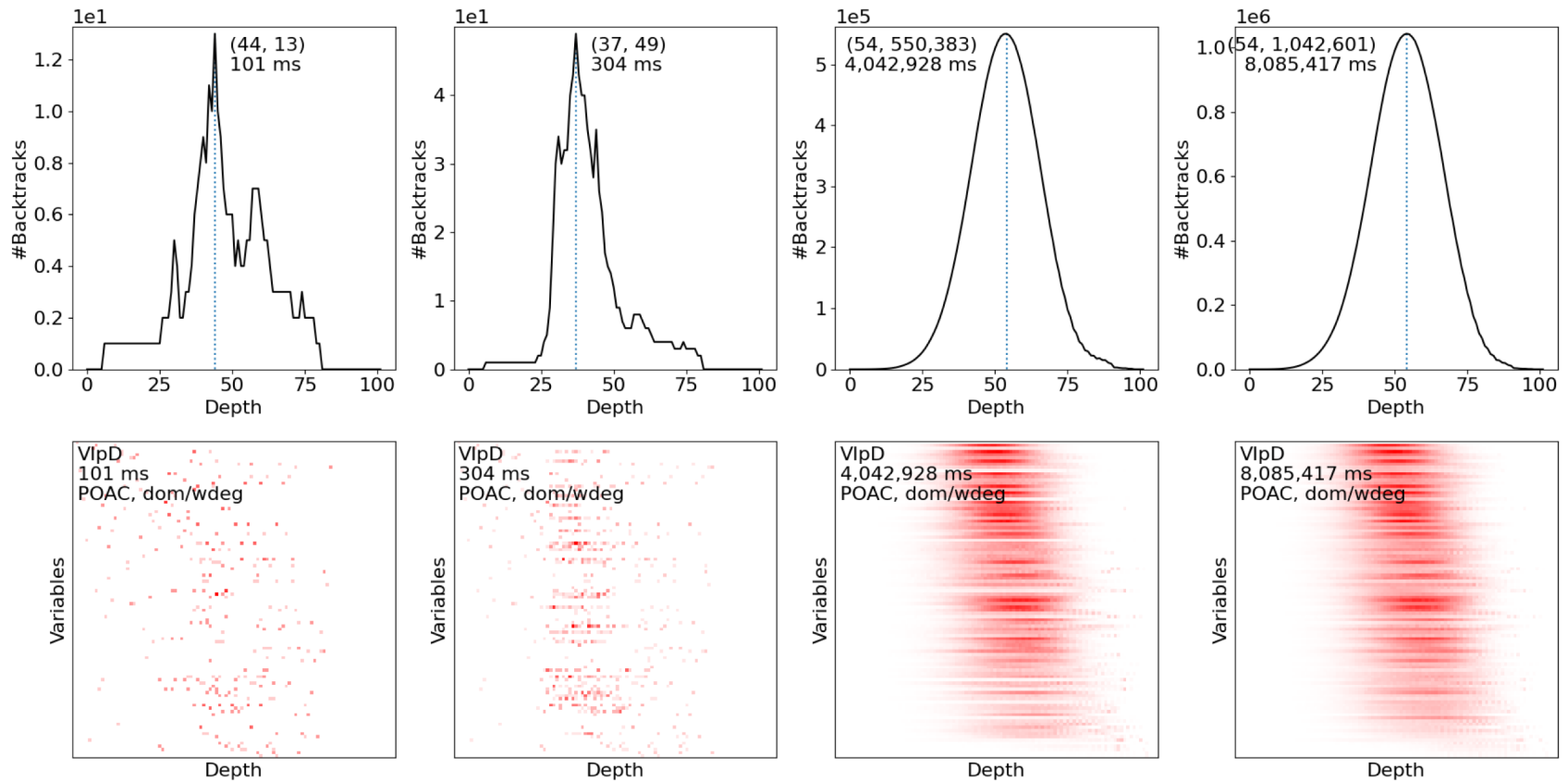
# Example: Variable ordering

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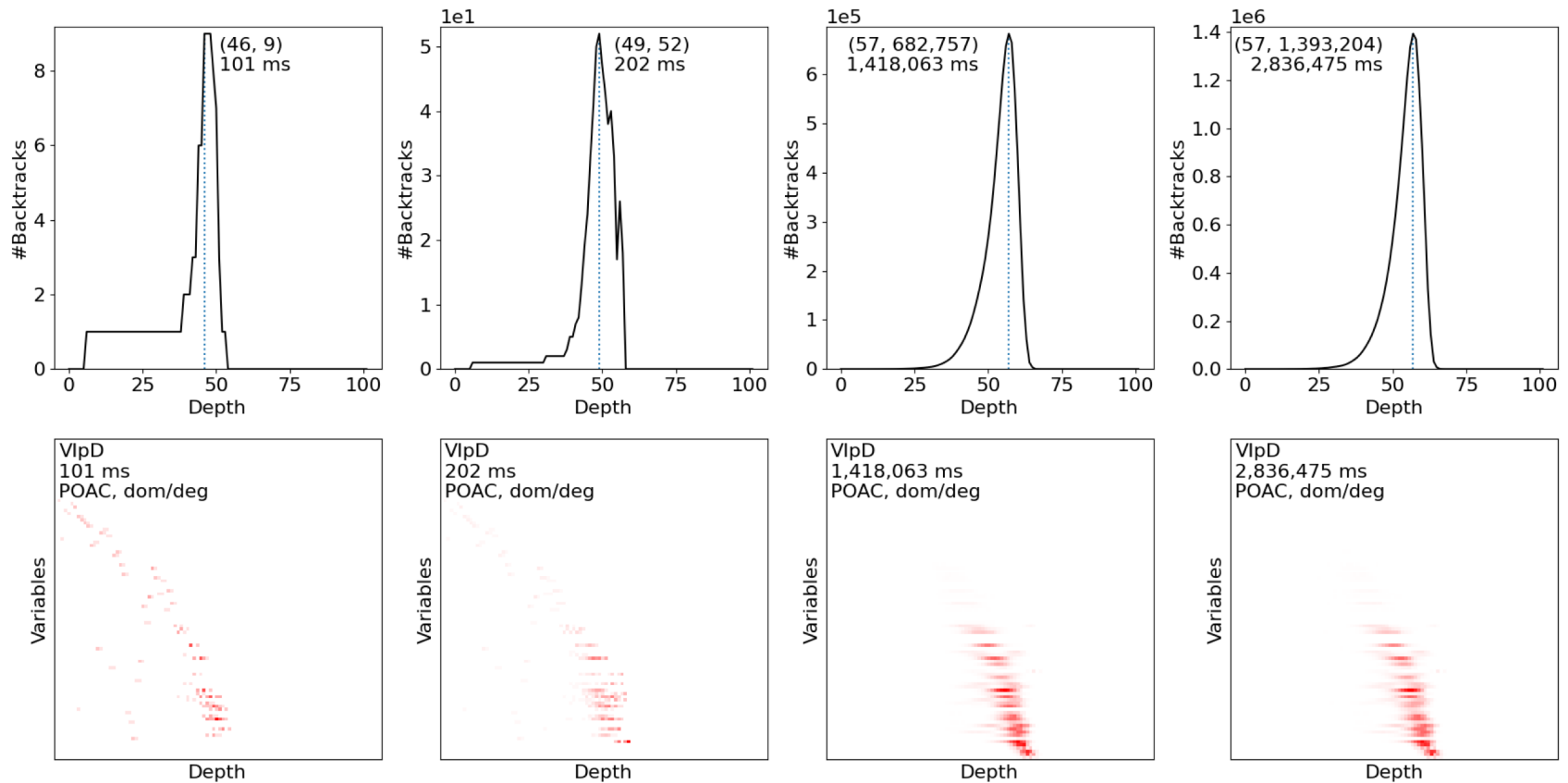
VIpD of POAC with dom/deg on MUG100-1-3

# Example: Variable ordering



0.1-summarization of POAC with dom/wdeg on MUG100-1-3

# Example: Variable ordering



0.1-summarization of POAC with dom/deg on MUG100-1-3

# Conclusions and discussion

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- Summarizations can be used to help explain the (changing) behavior of search
- Researchers and developers can use these tools iteratively to better study the impact of a strategy on a given problem
- Summarizations can catch small and large behavior that a human could not

# Conclusions and discussion

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- Initial results of BpD with binary branching give similar results
- We currently provide a ‘post-mortem’ analysis. Future work includes an ‘in-vivo’ analysis that enables human experts



# Acknowledgements

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